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ANALYTICAL MODAL ANALYSIS FOR THIN-FILM FLAT LENSES

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Introduction

Due to strong potential applications and more demanding requirements imposed upon thin-film structures for space deployable, there has been increasing research and development activities during recent years in the field of vibration analysis of these types of structures. Moreover, interests in employing these structural components have received renewed emphasis in recent years within NASA and the Air Force. This is due to their inherent lightweight, low packaging and launch volume, and relative simplicity of deployment. Among the potential mission concepts for which these structural elements are included, one can mention solar sails, space solar power generation systems, solar thermal propulsion vehicles, large space telescopes, and inflatable communication antennas.

This paper presents analytical procedures to determine vibration and physical characteristics of thin film lenses with circular and elliptical shapes membranes considered in design of a solar concentrator. In general, three methods are used to obtain approximate solutions of Helmholtz boundary value problems. One method requires that solution satisfy the differential equation exactly and the boundary condition approximately. Another method demands a solution that satisfies the boundary conditions exactly and the governing equations approximately. The third method sees a solution that satisfies both the governing equation and boundary conditions approximately. Extensive reviews of vibrations of membrane and plates are provided by Leissa (1965) and Mazumdar (1982). Figure 1 illustrates a solar concentrator assembly in a test facility with lens and thin-film struts.

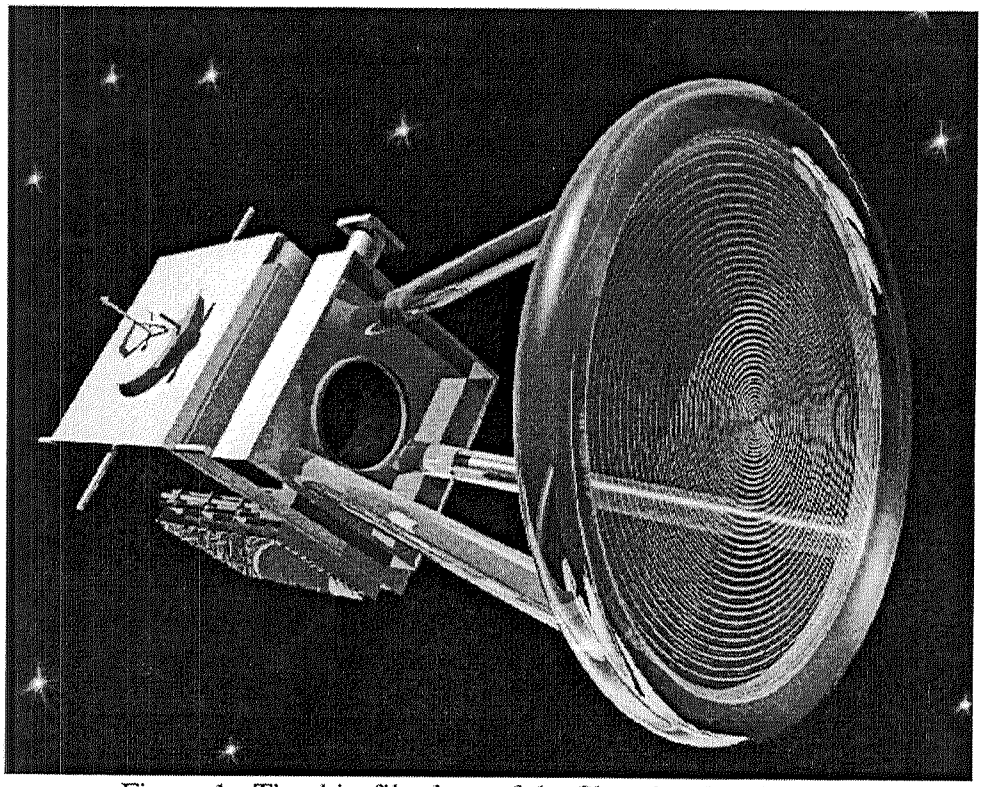

Figure 1. The thin-film lens of the Shooting Star Project

Vibrations Analysis of Circular Thin-Film Clamped Lens

Free vibration of a thin film circular lens made of CP1 supported by an aluminum ring is considered. Two different analytical models based on pre-strained membrane and plate theory are developed and the results are compared with experimental modal analysis. Comparing the results of experiment and membrane theory provides estimation for the pre-strain in the film. CP1 material and physical properties are: Film Thickness of 0.0005 in, Film Diameter of 17.5 in, Mass density of 0.0001341 lb s^2 /in^4 , Poisson's ratio of 0.34, and modulus of elasticity of 300000 psi. The mathematical model for transverse free vibration of the membrane as shown by Soedel (1993) can be represented by the following Helmholtz equation.

$$\nabla^2 w - \frac{\rho h}{N}\ddot{w} = 0 \tag{1}$$

Where ρ is the mass density, h is the film thickness, and N is the radial uniform boundary tension per unit length. Assuming harmonic vibration, the modal displacement function in polar coordinate can be written as:

$$w_n(r,\theta,t) = W_n(r)\cos n(\theta - \alpha)e^{i\omega t} \tag{2}$$

The solutions to $W_n(r)$ for each value of the integer n are two linearly independent solutions of Bessel's equation. However, since W_n is always a finite value for $r \leq 0$ and is zero at the boundary of $r = a$, therefore the frequency equation can be given by

$$J_n(\lambda a) = 0 \tag{3}$$

where $\lambda^2 = \dfrac{\rho h \omega^2}{N}$

The above equation is the frequency equation and has infinite number of roots for each value of n. few of these roots are listed in the Table I. The natural frequencies of the membrane are given by

$$\omega_{mn} = \frac{(\lambda a)_{mn}}{a}\sqrt{\frac{N}{\rho h}} \tag{4}$$

Table I. Values for $(\lambda a)_{mn}$

m	n			
	0	1	2	3
0	2.404	5.520	8.654	11.792
1	3.832	7.016	10.173	13.323
2	5.135	8.417	11.620	14.796
3	6.379	9.760	13.017	16.224

Table II. Experimental Modal Information

Frequency - Hz	Loss factor
53.3	1.680-002
185.9	1.298e-002
356.0	0.072e-002

Integers m and n represent number of nodal circles and nodal diameter for each mode respectively. Radial Tension force along a unit length of the membrane in terms of uniform pre-strain can be presented by the following equation $N = Eh\varepsilon_0/(1-\nu)$. Where ε_0 is the value of radial pre-strain. Considering that zeros of the first Bessel functions for non-negative orders are always real, it can be concluded that for linear viscoelastic material the modal loss factor for each mode is the same as the material loss factor. Computed natural frequencies for first sixteen modes for different values of strains are computed and are presented in Figure 2. The results illustrate as the pre- strain increases natural frequencies for all modes increase too. Theses results are compared with those obtained from Finite Element analysis for the fundamental

modes. The comparison establishes an excellent agreement between analytical and Finite Element results.

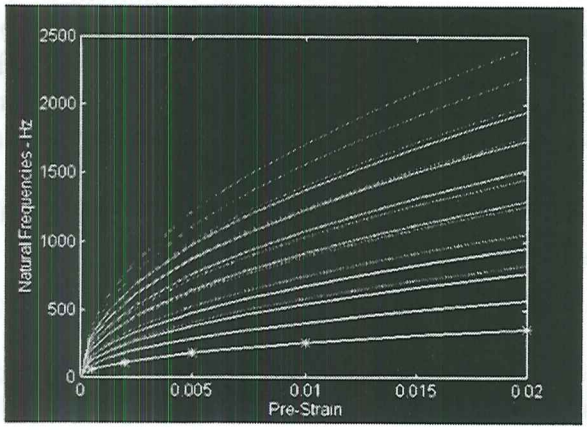

Figure 2. Natural frequencies versus pre-strain

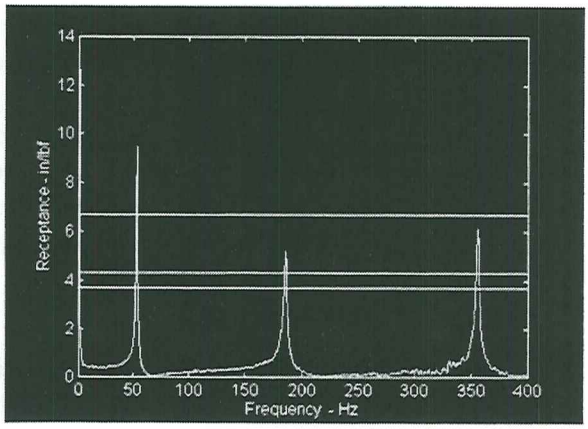

Figure 3. Experimental receptance response

Experimental Modal Analysis for a Circular Membrane

Experimental modal analysis was also conducted on CP1 membranes with the same geometry and properties. The tests were performed using miniature impulse hammer for excitation while laser vibrometer was used to measure the response of the membrane. Frequency responses for excitation and response measurement at center and half way along the radius were determined. For the case of driving and measuring points to be at the center, the receptance versus frequencies is plotted in Figure 3. Using Half power point technique natural frequencies and modal loss factors for the three major peaks shown in figure 3 are tabulated in Table II. Natural frequencies for fifteen different modes for several pre-strain values were computed and were compared with those obtained experimental with measured point being halfway along the radius. Figure 4 shows that the comparison established good agreement between analytical results for pre-strain of 0.0004 and the experimental data. This comparison can be used to estimate the amount of pre-strain and consequently the radial tension per unit length of the circular membrane.

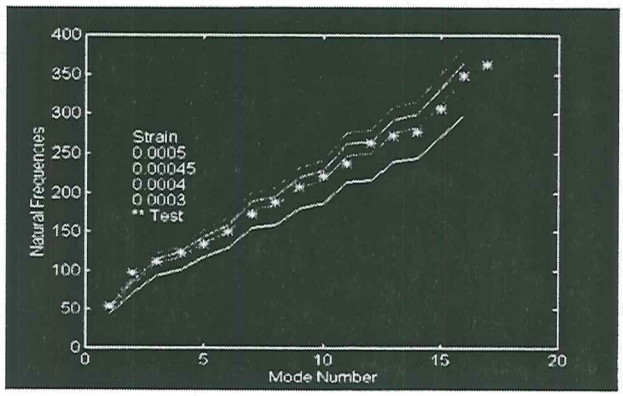

Figure 4. Natural frequencies for different pre-strains

Free Vibration of Clamped Thin Film Elliptical Membrane

Free vibration of a thin film elliptical n XXI-4 ıade of CP1 supported at its boundary is considered. An analytical model is developed and a solution based on Mathieu functions in elliptical coordinate is provided. In the analysis following material properties for CP1 material are assumed. Film Thickness of t = 0.0005 in, Film major diameter of a = 226 in, Film minor diameter b= 164.08 in, Mass density of ρ = 0.0001341 lb s^2 /in^4, Poisson's ratio of ν = 0.34, Modulus of elasticity of E = 300000 psi. The mathematical model for transverse free vibration of the membrane can be represented by the following Helmholtz equation

$$\nabla^2 w - \frac{\rho h}{N}\ddot{w} = 0 \tag{5}$$

Where ρ is the mass density, h is the film thickness, and N is the radial uniform boundary tension per unit length. Assuming harmonic vibration, the modal displacement function in polar coordinate can be written as:

$$w(x, y, t) = W(x, y)\, e^{i\omega t} \tag{6}$$

transforming from Cartesian to elliptical coordinates on may write:

$$x + i\,y = h\cosh(\zeta + i\,\eta) \tag{7}$$

Transforming the Laplacian operator from Cartesian to the elliptical coordinate after extensive simplifications the equation of motion will be simplified by introducing a separable solution of $W(\zeta, \eta) = \Psi(\zeta)\,\phi(\eta)$ in elliptical coordinate. Further simplification results in getting two ordinary differential equations called Mathieu and modified Mathieu equations (McLachlan (1947)).

$$\frac{d^2\phi}{d\eta^2} + (a - 2k^2\cos 2\eta)\phi = 0 \quad \text{and} \quad \frac{d^2\psi}{d\zeta^2} - (a - 2k^2\cosh 2\zeta)\psi = 0 \tag{8}$$

After satisfying all the required boundary conditions, solution of $w(\zeta, \eta)$ which closely satisfy the above conditions is:

$$w(\zeta,\eta) = \sum_{m=0}^{\infty} C_m Ce_m(\zeta, q)ce_m(\eta, q)\cos(\omega_m t + \gamma) + \sum_{m=0}^{\infty} S_m Se_m(\zeta, q)se_m(\eta, q)\cos(\bar{\omega}_m t + \bar{\gamma}) \tag{9}$$

Where $C_m, S_m, \gamma_m, and\ \bar{\gamma}_m$ are arbitrary constant determined from the initial conditions, $\omega_m and\ \bar{\omega}_m$ are natural frequencies of the mth mode. Each individual solution in the above equation for m = 0, 1, 2, ... corresponds to a different modes of vibration. Where, q has its appropriate value. Considering the above conditions we must have

$$Ce_m(\zeta_0, q) = 0 \ \ for\ m = 0,1,2,... \ and \ \ Se_m(\zeta_0, q) = 0 \ \ for\ m = 1,2,... \tag{10}$$

The value of ζ_0 is fixed, so we need those positive values of q, say $q_{m,r}, \bar{q}_{m,r}$ for which for which the above two functions vanish. These may be regarded as the positive parametric zeros of the functions. There is an infinity of zeros for each m. Hence for Ce_m if r = p, there are (p-1) nodal ellipses within the clamping rings, and likewise for Se_m , but their location differ. Natural frequencies are related to $q_{m,r}, \bar{q}_{m,r}$. For $0 \le q$, $1 \le m$, the functions ce_m (η,q), se_m (η,q) have zeros in η. Hence w vanishes also if η satisfies the respective equations

$$f_{m,r} = \frac{1}{\pi h}\sqrt{\frac{Nq_{m,r}}{\rho t}} \quad and \quad \bar{f}_{m,r} = \frac{1}{\pi h}\sqrt{\frac{N\bar{q}_{m,r}}{\rho t}} \tag{11}$$

$$ce_m(\eta, q_{m,r}) = 0 \, and \, se_m(\eta, \bar{q}_{m,r}) = 0 \tag{12}$$

Now $ce_m(\eta, q_{m,r})$ *and* $se_m(\eta, \bar{q}_{m,r})$ each have m zeros in $0 \le \eta < \pi$, so for a given m each function rise to m nodal hyperbolas. Therefore, these roots define a series of confocal nodal hyperbolas. In this analysis, approximate functions for Computation of Ce_m, Se_m, ce_m, and se_m given by McLachlan (1954) are adopted. Of course, the accuracy of the numerical values of natural frequencies is entirely depends on the accuracy of these functions. For the given elliptical membrane with major and minor diameters of 226 in and 164.08 in respectively we have h = 155.41 in and the eccentricity of the ellipse is $e = h/a = 0.6877$. since cosh $(\zeta_0) =$ 1/e, therefore is $\zeta_0 = 0.9203$. first roots of $Ce_m(\zeta_0, q)=0$ and $Se_m(\zeta_0, q)=0$ for m = 0, 1, and 2 are computed to be

$q_{01} = 0.9967$, $q_{11} = 2.0506$, q_{11} = is complex, $q_{21} = 3.2729$, and $q_{21} = 3.9323$

Considering pre-strain of $\varepsilon_0 = 0.03\%$ value of pre-tension force per length of membrane will be calculated to be N = 0.06818 lb/in.

Using equation (11) estimated natural frequencies in Hz for the elliptical membrane are obtained to be:

$f_{01} = 4.1238$, $f_{11} = 5.9150$, $\bar{f}_{11} = 7.044$, $f_{21} = 7.4728$, and $\bar{f}_{21} = 8.1911$

Conclusion

The following conclusions are drawn from the present study:

1. Analytical models for solutions of linear free vibrations of circular and elliptical membranes are developed.
2. The solutions provide the natural frequencies and mode shapes for these clamped membranes. Excellent comparison, between the analytical and experimental results, concludes that the linear model is adequate for the required analysis.
3. Analysis has demonstrated that experimental results for natural frequencies can be used to determine the radial tension force within the membrane.
4. Pre-stress due to the tension in the membrane has a marked effect on the flexural vibrations of both types of membranes.

References

[1] Leissa, A. W., "Vibration of plates," *NASA SP-288,* 1969.

[2] Mazumdar, J., "A review of approximate methods for determining the vibrational modes of membranes," Shock and Vibration Digest, Vol. 14, pp. 11-17, 1982.

[3] McLachlan, N. W, "Vibrational problems in elliptical coordinates," Quart. Appl. Math., 1947

[4] McLachlan, N. W, "Theory and application of Mathieu functions," Dover publishing Company, 1954.

[5] Soedel, W., "Vibrations of shells and plates," 2nd Edition, Marcel Dekker, 1993.